KEK-TH-478
KEK preprint 96-5
April 1996
H

$\mu \to e\gamma$ Search with Polarized Muons

Yoshitaka Kuno and Yasuhiro Okada

Department of Physics, National Laboratory for High Energy Physics (KEK),

Tsukuba, Ibaraki, Japan 305

Abstract

A search for the lepton-flavor violating $\mu^+ \to e^+\gamma$ decay using polarized muons is proposed. By measuring the angular distribution of e^+s with respect to the muon spin direction, in particular antiparallel e^+s, the serious physics background from $\mu^+ \to e^+\nu\overline{\nu}\gamma$ as well as accidental background from normal muon decay accompanied by a high-energy photon can be suppressed significantly. In addition to the enhancement of the sensitivity, the angular distribution would discriminate among different extensions to the Standard Model, once the signal is observed.

PACS numbers: 13.35.Bv, 11.30.Fs, 13.88.+e, 12.60.Jv,

Lepton flavor violation (LFV) has been attracting much attention recently, since several extensions to the Standard Model predict large LFV rates. Among the various theoretical models with LFV, models of supersymmetric grand unification (GUT) in particular predict a branching ratio only one or two order magnitudes lower than the current experimental limits [1,2]. A search for LFV in low energy processes would have the potential to test unification at very high energy. Compared to other LFV processes of interest such as $\mu^+ \to 3e$, $\mu^- - e^-$ conversion in a nucleus, $\tau \to \mu\gamma$, etc. $\mu^+ \to e^+\gamma$ is known to have a higher sensitivity to supersymmetric unification [2].

The previous experiments searching for $\mu^+ \to e^+\gamma$ have been done with a surface muon beam of low-energy (4 MeV) positive muons from pions stopped in the pion production target. Due to their production mechanism, surface muons are originally 100 % polarized, antiparallel to their flight direction. However, none of the past experiments used the polarization, but rather depolarized the muons by using, for instance, a polystyrene muon-stopping target [3]. Here, the importance of the muon polarization to search for $\mu^+ \to e^+\gamma$ will be presented. For instance, the measurement of the angular distribution of $\mu^+ \to e^+\gamma$ with respect to the muon-polarization direction would give information to discriminate among various models, since different theoretical models predict a different helicity of e^+ in $\mu^+ \to e^+\gamma$, as shown below.

Supersymmetric unified theories predict large rates for LFV processes such as $\mu^+ \to e^+\gamma$ as a consequence of the large top-quark Yukawa coupling. The SU(5) supersymmetric (SUSY) GUT model predicts a branching ratio between 10^{-15} to 10^{-13} for the singlet selectron mass of $m_{\tilde{e}_R}$ of 100 to 300 GeV, and the SO(10) SUSY-GUT models give an even larger value of 10^{-13} to 10^{-11} [2]. In these supersymmetric unified models, interactions at the GUT energy scale could induce lepton flavor mixing in the left-handed and/or right-handed slepton sectors. As a consequence, LFV processes such as $\mu \to e\gamma$ occur at a low energy, and they therefore carry information on the interactions at the GUT energy scale. The helicity of e^+ in $\mu^+ \to e^+\gamma$ is subject to the mechanism of flavor mixing in the slepton sectors. For instance, the minimal SU(5) SUSY-GUT model introduces a lepton flavor mixing only

on the right-handed slepton sector, $\tilde{e}_R$, and therefore, only $\mu^+ \to e^+_L\gamma$ occurs, where e^+_L is a left-handed positron. On the other hand, the SO(10) SUSY-GUT models have a lepton flavor mixing on the $\tilde{e}_L$ sector as well as the $\tilde{e}_R$ sector, giving rise to $\mu^+ \to e^+_R\gamma$ as well as $\mu^+ \to e^+_L\gamma$. Furthermore, some non-unified supersymmetric extensions to the Standard Model with heavy right-handed neutrinos also expect a large branching ratio of orders of 10^{-13} to 10^{-11} [4] and $\mu^+ \to e^+_R\gamma$ is predicted. Some non-supersymmetric extensions to the Standard Model, such as left-right symmetric models and extra Higgs models, also predict a sizeable branching ratio of $\mu^+ \to e^+\gamma$ [5].

However, to improve sensitivity and measure angular distribution, major background decays to a search for $\mu^+ \to e^+\gamma$, in particular their angular distribution with respect to the muon-polarization direction, have to be investigated. Those backgrounds are a physics background from the radiative muon decay $\mu^+ \to e^+\nu\overline{\nu}\gamma$, and an accidental background of the normal muon decay $\mu^+ \to e^+\nu\overline{\nu}$ accompanied by a high energy photon. In this letter, we propose the use of polarized muons to search for $\mu^+ \to e^+\gamma$ to improve sensitivity against those backgrounds, and especially emphasize the importance of the selective measurement of e^+s going opposite to the muon-polarization direction, which would reduce the backgrounds significantly and allow the measurement with a higher sensitivity in future experiments.

In general, the Lagrangian for $\mu^+ \to e^+\gamma$ decay can be given [6] with an explicit helicity expression by

$$\mathcal{L} = A_R\ \overline{\mu}_R\sigma^{\mu\nu}e_L F_{\mu\nu} + A_L\ \overline{\mu}_L\sigma^{\mu\nu}e_R F_{\mu\nu} + h.c. \tag{1}$$

where $\mu_L(\mu_R)$ is a left-handed (right-handed) muon and $e_L(e_R)$ is a left-handed (right-handed) electron. A_R and A_L are coupling constants. $F_{\mu\nu}$ is $\partial_\mu A_\nu - \partial_\nu A_\mu$ and A_μ is an electromagnetic field. For polarized muons, from Eq.(1), the angular distribution of e^+ in $\mu^+ \to e^+\gamma$ with respect to the direction of μ^+ spin direction can be given by

$$d\Gamma(\mu^+ \to e^+\gamma) = \frac{1}{8\pi}(1-\frac{m_e^2}{m_\mu^2})^3 m_\mu^3 d(\cos\theta) \times \left[|A_R|^2(1-P_\mu\cos\theta) + |A_L|^2(1+P_\mu\cos\theta)\right] \tag{2}$$

where θ is the angle between the muon spin and the e^+ direction. P_μ is the muon spin polarization, which for surface muons is almost 100 %. m_μ and m_e are the muon mass and

the positron mass, respectively. The minimal SU(5) SUSY-GUT model predicts a non-zero A_L and a vanishing A_R, yielding a $(1+P_\mu \cos\theta)$ distribution. On the other hand, the simplest version of the SO(10) SUSY-GUT models predict approximately-equal helicity amplitudes for right-handed and left-handed e^+s ($A_L \approx A_R$) [2], resulting in an almost uniform angular distribution. For some non-unified supersymmetric models [4], A_L is vanishing but A_R is non-zero, giving a $(1 - P_\mu \cos\theta)$ distribution. Therefore, the measurement of the angular distribution of e^+ with respect to the direction of muon polarization would provide a means to discriminate among these models clearly when the signals are observed.

The event signature of $\mu^+ \to e^+\gamma$ is that the energies of both e^+ and photon are equal to a half of the muon mass ($m_\mu/2 = 52.8$ MeV) and they are collinear back to back. The current experimental upper limit is 4.9×10^{-11} at 90 % confidence level [3].

The major physics backgrounds to the search for $\mu^+ \to e^+\gamma$ decay is radiative muon decay, $\mu^+ \to e^+\nu\overline{\nu}\gamma$ (branching ratio = 1.4 % for $E_\gamma > 10$ MeV). When the e^+ and photon are emitted back-to-back with two neutrinos carrying off little energy, it becomes a serious physics background to $\mu^+ \to e^+\gamma$. The differential decay width of this radiative decay was calculated as a function of e^+ energy (E_e) and photon energy (E_γ) normalized to their maximum energies (of $m_\mu/2$), namely $x = 2E_e/m_\mu$ and $y = 2E_\gamma/m_\mu$, where x and y range from 0 to 1 [7,8]. As a background to $\mu^+ \to e^+\gamma$, the kinematic case when $x \approx 1$ and $y \approx 1$ is important. Its angular distribution has to be examined especially. In an approximation of the limit of $x \approx 1$ and $y \approx 1$ with an angle between e^+ and photon ($\theta_{e\gamma}$) of almost 180°, the differential decay width of $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay is given by

$$\begin{aligned} &d\Gamma(\mu \to e\nu\overline{\nu}\gamma) \cong \frac{G_F^2 m_\mu^5 \alpha}{3 \times 2^8 \pi^4} \times \\ &\Big[(1-x)^2(1 - P_\mu \cos\theta) + \Big(4(1-x)(1-y) - \frac{1}{2}z^2\Big)(1 + P_\mu \cos\theta)\Big] dx dy z dz d(\cos\theta) \qquad (3) \end{aligned}$$

where G_F is the Fermi coupling constant, α is the fine-structure constant, $z = \pi - \theta_{e\gamma}$, and $\cos z$ is expanded in a polynomial of z since z is small. In Eq.(3), only the terms of up to the second order of a combination of $(1-x)$, $(1-y)$ and z are kept. At $x \approx 1$ and $y \approx 1$, the effect of the positron mass is found to be very small, of order $(m_e/m_\mu)^2$, and therefore

neglected in Eq.(3). The first term in Eq.(3) represents the e^+ being emitted preferentially opposite to the muon spin direction, whereas in the second term the e^+ is emitted along the muon-spin direction. When $x = 1$ and $y = 1$ exactly, this differential decay width vanishes. But in a real experiment, finite detector resolutions introduce background events which would ultimately limit the sensitivity of a search for $\mu^+ \to e^+\gamma$.

Given the detector resolutions, the sensitivity limitation from $\mu^+ \to e^+\nu\overline{\nu}\gamma$ background decay can be evaluated by integrating the differential decay width in Eq.(3) over the resolutions, or more precisely, over the kinematic box region of the signal which is determined by the detector resolutions [9]. Take δx, δy and δz to be the kinematic range of the signal region for e^+ energy $(1 - \delta x \leq x \leq 1)$, that for photon energy $(1 - \delta y \leq y \leq 1)$ and that for the angle of $z = \pi - \theta_{e\gamma}$ $(0 \leq z \leq \delta z)$, respectively. The integration was done with consideration of the kinematics constraints among x, y and z. Namely, if δx and δy are small, the allowed range of z is determined to be $0 \leq z \leq 2\sqrt{(1-x)(1-y)}$, instead of δz. The partial branching ratio after the integration is given by

$$\begin{aligned} dB(\mu \to e\nu\overline{\nu}\gamma) &= \frac{1}{\Gamma(\mu \to e\nu\overline{\nu})} \int_{1-\delta x}^{1} dx \int_{1-\delta y}^{1} dy \int_{0}^{\min(\delta z, 2\sqrt{(1-x)(1-y)})} dz \;\; \frac{d\Gamma(\mu \to e\nu\overline{\nu}\gamma)}{dxdydz} \\ &= \frac{\alpha}{16\pi}\Big[J_1(1 - P_\mu \cos\theta) + J_2(1 + P_\mu \cos\theta)\Big] d(\cos\theta), \end{aligned} \tag{4}$$

where $\Gamma(\mu \to e\nu\overline{\nu})$ is the total muon decay width. For the case of $\delta z > 2\sqrt{\delta x \delta y}$, J_1 and J_2 are given by

$$J_1 = (\delta x)^4(\delta y)^2 \quad \text{and} \quad J_2 = \frac{8}{3}(\delta x)^3(\delta y)^3, \tag{5}$$

i.e., they can be reduced as the sixth power of a combination of δx and δy. In Fig.1, the sensitivity limit imposed by the $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay with unpolarized muons is shown. From Fig.1, it can be seen that to achieve an sensitivity limit of a level of 10^{-15}, both δx and δy of an order of 0.01 are needed.

Experimentally, improvement of the photon-energy resolution could be more difficult than that of the e^+ energy resolution. For instance, in the MEGA experiment which aims to measure the branching ratio sensitive to a level of 6×10^{-13} [10], the proposed numbers

of the detector resolutions are $\triangle E_e = 0.4$ MeV (FWHM) but $\triangle E_\gamma = 1.4$ MeV (FWHM). It is likely that the improvement of photon energy resolution is limited. Assuming that δy is worse than δx by a factor of several, together with the factor of 8/3, J_2 in Eq.(4) could be an order magnitude larger than J_1. Therefore, the distribution follows mostly $(1+P_\mu \cos\theta)$ as long as $\delta y > \delta x$. Fig.2 shows the angular distribution of $\mu^+ \to e^+\nu\overline{\nu}\gamma$ with for instance $\delta y/\delta x = 4$. It implies that if we measure selectively e^+s in $\mu^+ \to e^+\gamma$ going opposite to the muon-polarization direction, the background from $\mu^+ \to e^+\nu\overline{\nu}\gamma$ will be reduced significantly. Furthermore, by varying δx and δy, the angular distribution of the $\mu^+ \to e^+\nu\overline{\nu}\gamma$ background can be changed according to Eq.(4), thus providing another means to discriminate the signal from the backgrounds.

When the angular resolution of δz is better than the kinematically-allowed angle of $2\sqrt{\delta x \delta y}$, the distribution is given in Eq.(4) by

$$\begin{aligned} J_1 &= \frac{8}{3}(\delta x)^3(\delta y)(\frac{\delta z}{2})^2 - 2(\delta x)^2(\frac{\delta z}{2})^4 + \frac{1}{3}\frac{1}{(\delta y)^2}(\frac{\delta z}{2})^8 \\ J_2 &= 8(\delta x)^2(\delta y)^2(\frac{\delta z}{2})^2 - 8(\delta x)(\delta y)(\frac{\delta z}{2})^4 + \frac{8}{3}(\frac{\delta z}{2})^6 \end{aligned} \tag{6}$$

Similarly to Eq.(5), J_2 is larger than J_1 by roughly a factor of $\delta y/\delta x$. The above argument is also valid in this case.

Another serious background to $\mu^+ \to e^+\gamma$ is an accidental coincidence of a high-energy e^+ in the normal muon decay, $\mu^+ \to e^+\nu\overline{\nu}$, with a high-energy photon, likely from $\mu^+ \to e^+\nu\overline{\nu}\gamma$ [3,10]. This accidental background can be reduced in principle by an improved timing resolution of the experimental apparatus. However, at a high event rate, it is desirable to reduce this background by some additional means, and the use of polarized muons turns out to be very effective. The high-energy e^+ in the normal muon decay ($x \approx 1$) is known to be emitted preferentially along the muon spin direction, following an angular distribution of $(1 + P_\mu \cos\theta)$. When e^+s going opposite to the muon-spin direction are selected, this background is also reduced.

Thus, all the serious backgrounds to $\mu^+ \to e^+\gamma$ decay are expected to follow a $(1 + P_\mu \cos\theta)$ angular distribution of for e^+s. The detection of e^+s emitted opposite to the

muon-spin direction would eliminate backgrounds, whereas the acceptance of the signals for $\mu^+ \to e_R^+\gamma$ decay, which follows a $(1 - P_\mu \cos\theta)$ distribution, is kept high. However, the sensitivity to $\mu^+ \to e_L^+\gamma$ will be the same as the case with unpolarized muons. The angular distributions of $\mu^+ \to e_R^+\gamma$ and $\mu^+ \to e_L^+\gamma$ are also shown in Fig.2. In the design of future experiments, the solid-angle acceptance must be optimized with consideration of the required background rejection and the available muon beam intensity. For instance, taking a solid angle coverage of the detection of 45° (60°) in polar angle with respect to the direction antiparallel to the muon polarization would give an improvement in a signal-to-background ratio of about 12 (about 7), with a reasonable acceptance. Given a future increase in muon-beam intensity, a next-generation experiment measuring the branching ratio sensitive to a level of $10^{-14} - 10^{-15}$ without any backgrounds could be possible with polarized muons.

Since a surface muon beam is already polarized to 100%, the measurement with polarized muons will require a suitable material as a muon-stopping target which preserves the muon spin. Further, since the direction of muon polarization is antiparallel to its flight direction, the installation of a muon-spin rotator [11] which rotates the muon-spin direction perpendicular to the beam direction can be considered for more convenient detector arrangement.

In conclusion, a search for $\mu^+ \to e^+\gamma$ decay with polarized muons has the potential for improved sensitivity. The selective measurement of e^+s going antiparallel to the muon-spin direction would reduce the physics backgrounds from $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay as well as from the accidental background of the normal muon decay accompanied by a high-energy photon. In addition, when the signal is observed, its angular distribution would give a clear discrimination of models and a significant test of supersymmetric unification.

ACKNOWLEDGMENTS

The authors are grateful to Dr. J.A. Macdonald for the information on a muon-spin rotator. This work was supported in part by the Grant-in-Aid of the Ministry of Education,

Science, Sports and Culture, Government of Japan.

REFERENCES

[1] L.J. Hall, V.A. Kostelecky and S. Raby, Nucl. Phys. **B267**, 415 (1986).

[2] R. Barbieri and L. Hall, Phys. Lett. B **338**, 212 (1994);
R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. **B445**, 219 (1995).

[3] R.D. Bolton *et al.*, Phys. Rev. D **38**, 2077 (1988).

[4] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986);
J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B **357**, 579 (1995);
J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53**, 2442 (1996);
R. Rattazzi and U. Sarid, preprint UND-HEP-95-US01, RU-95-72, hep-ph/9512354, 1995.

[5] see for instance, a review by J.D. Vergados, Phys. Rep. **133**, 1 (1986);
R.N. Mohapatra, Phys. ReV. D **46** 2990 (1992);
D. Chang, W.S. Hou, and W.-Y. Keung, preprint CERN-TH.6795/93, NUHEP-TH-92-26, NTUTH-92-24, hep-ph/9302267, 1993.

[6] S. Weinberg and G. Feinberg, Phys. Rev. Lett. **3**, 111 (1959).

[7] C. Fronsdal and H. Überall, Phys. Rev. **118**, 654 (1959).

[8] S.G. Eckstein and R.H. Pratt, Ann. of Phys. **8**, 297 (1959).

[9] The size of the signal region is generally assigned by a few times r.m.s of the detector resolution in order to keep a high acceptance of the signal.

[10] E.V. Hungerford, in *Proceedings of the 23rd INS International Symposium on Nuclear and Particle Physics with Meson Beams in the 1 GeV/c Region, Tokyo, 1995*, edited by S. Sugimoto and O. Hashimoto (Universal Academy Press Inc., 1995) p.131; J.J. Szymanski, in *Proceedings of International Conference on Intersections between Particle*

and Nuclear Physics, St. Petersburg, 1994, edited by S.J. Seestrom (AIP Conference Proceedings 338, 1994) p.789.

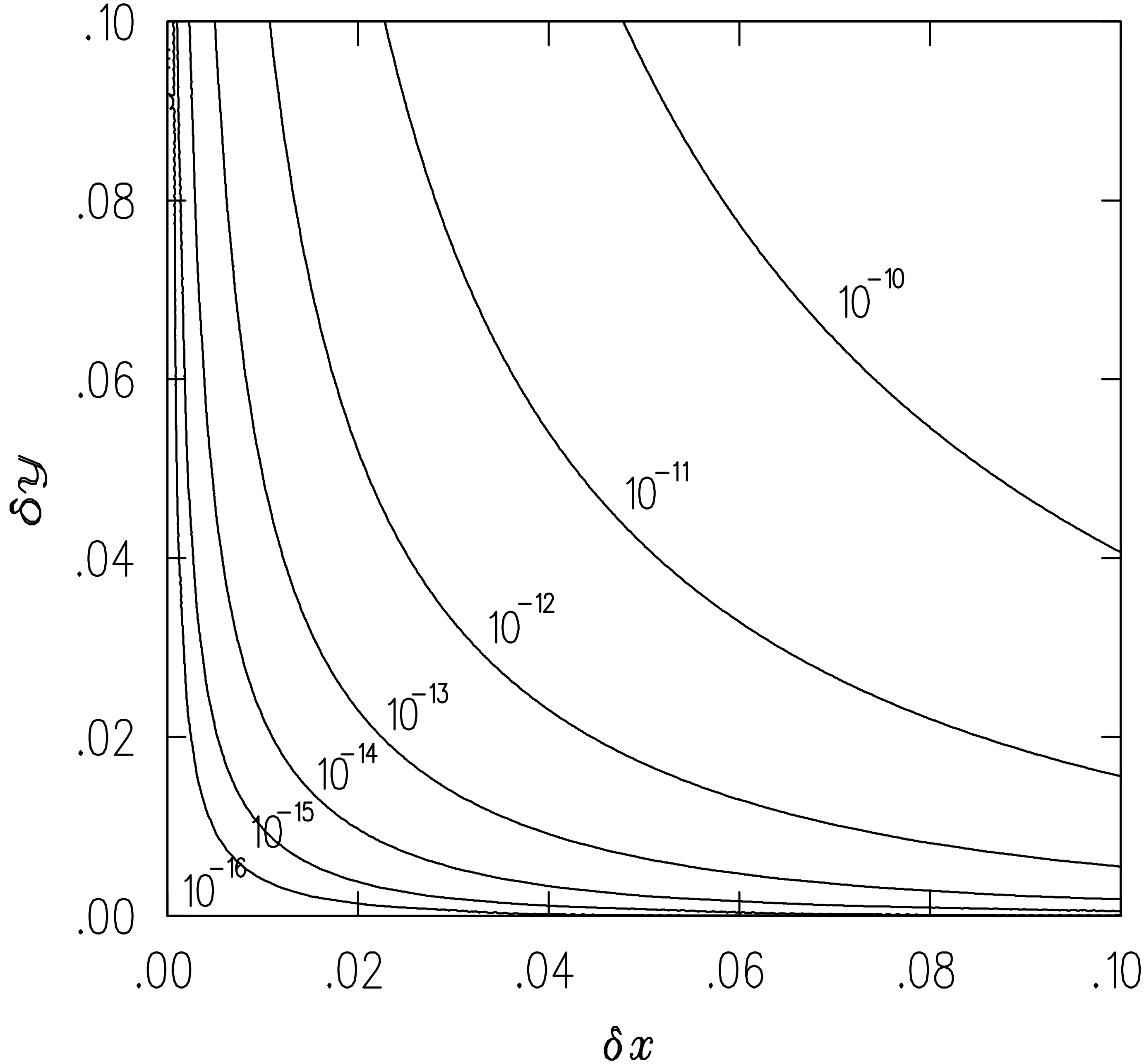
[11] See, for instance, a report on the M15 channel at TRIUMF, G. M. Marshall, Z. Phys. C - Particles and Fields **56**, S226 (1992).

FIGURES

FIG. 1. Sensitivity limitation of the branching ratio of $\mu^+ \to e^+\gamma$ imposed by $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay for the case of unpolarized muons as a function of δx and δy.

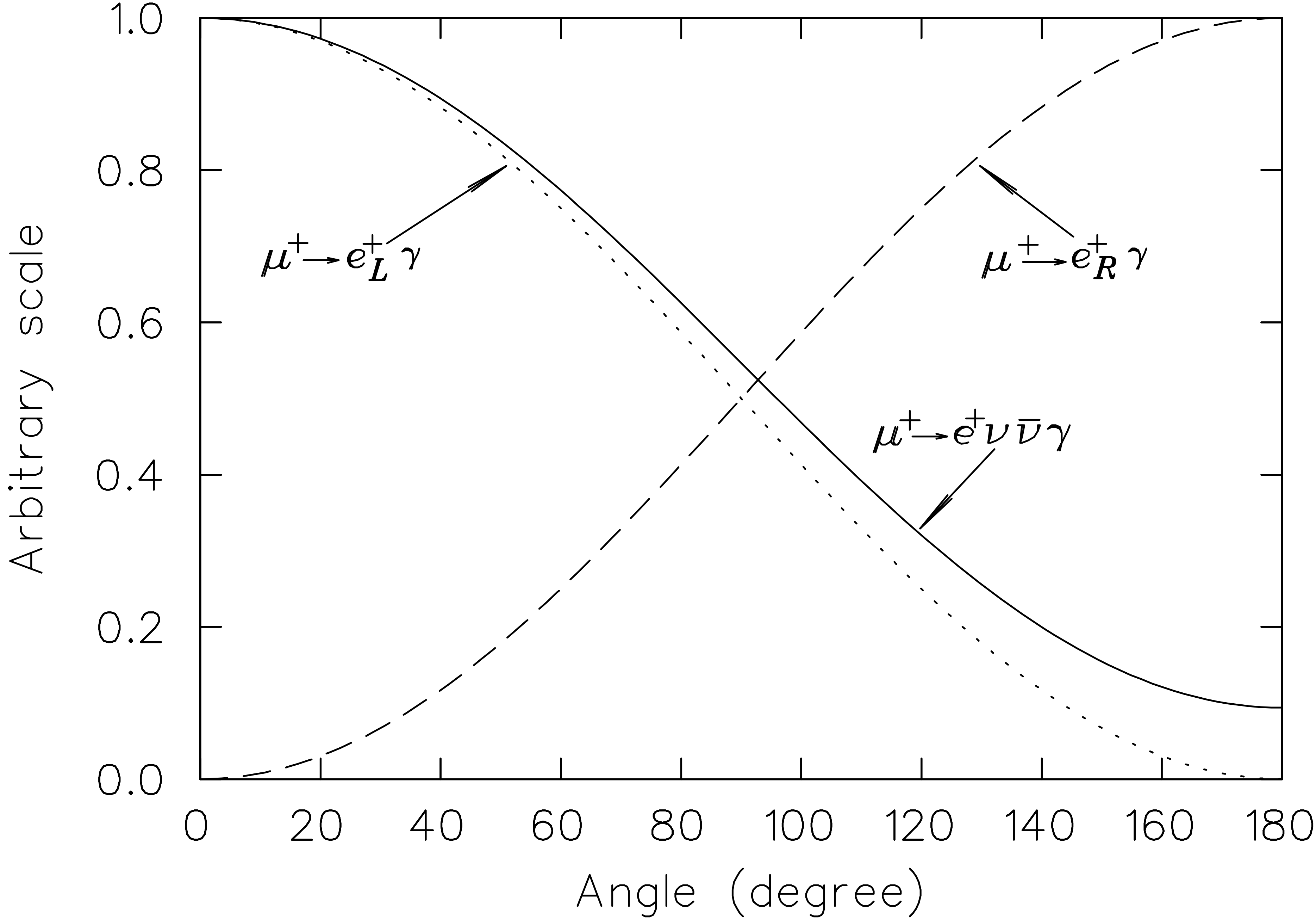
FIG. 2. Angular distribution of e^+ in $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay with $\delta y/\delta x = 4$ as a function of angle from the muon-polarization direction in a solid line. A dot line and dash line show that of $\mu^+ \to e^+_L\gamma$ and $\mu^+ \to e^+_R\gamma$ decay, respectively. A vertical scale is arbitrary.

Fig.1



.10
.08
.06
.04
.02
.00
δy
.00
.02
.04
.06
.08
.10
δx
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}

Fig.2



$\mu^+ \to e_L^+ \gamma$
$\mu^+ \to e_R^+ \gamma$
$\mu^+ \to e^+ \nu \bar{\nu} \gamma$
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
120
140
160
180
Arbitrary scale
Angle (degree)